Series – Semiconductor physics and light-matter interaction
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Series 1 – Dimensional analysis: Solutions

Exercise 1

a) The reduced Planck constant, $\hbar$, has the dimension of an energy × time, i.e., $[\hbar]=ML^2T^{-2}\times T$.
We can also express the energy, E, as a force × length and force = mass × acceleration (commonly expressed in Newton).

By also recalling that the wavevector, k, is the inverse of a length, one obtains:

$$[m^*] = ((ML^2T^{-2}\times T)^2\times L^{-2})/(ML^2T^{-2}) = M.$$

Hence, as expected m^* has the dimension of a mass, that is commonly expressed in kg when using SI units.

In the nearly-free electron model, the inverse of the second derivative of the energy band dispersion, $E(k)$, provides insight into the electron effective mass in the crystal.

b) By inserting the appropriate units, one obtains:

$$[1/(ML^2T^{-2}\times T)] \times [ML^2T^{-2}\times L] = LT^{-1},$$

which corresponds to a velocity. Thus, the study of the first derivative of the energy band dispersion, $E(k)$, can provide insight into the electron velocity in the crystal.

Exercise 2

a) We express ε_0 in $C^2.N^{-1}.m^{-2}$ and μ_0 in $N.A^{-2}$.
We recall that the Ampere (A) is the unit of the electric current and is expressed in $C.s^{-1}$.

One thus has:

$$[Z_0] = (((IT)^2\times(MLT^{-2})^{-1}\times L^{-2})/(MLT^{-2}\times I^{-2}))^{0.5} = ML^2T^{-3}I^{-2}.$$

Thus, Z_0 has the dimension of a resistance and is accordingly expressed in Ω. It is often described as the ratio of a voltage by a current (V/A).

By inserting the numerical values, one finds $Z_0 \sim 377$ Ω.

b) Expressing N in m^{-3}, q in C, ε_0 in $C^2.N^{-1}.m^{-2}$, m_0 in kg, ω and ω_0 in $rad.s^{-1}$, one gets:

$$[n_{op}{}^*] = ((IT)^2\times L^{-3})/((IT)^2\times(MLT^{-2})^{-1}\times L^{-2}) \times M\times T^{-2})=1,$$

which is a dimensionless quantity.

c) Using the fact that $[Z_0] = ML^2T^{-3}I^{-2}$ one gets:

$$[X] = ((ML^2T^{-3}I^{-2})\times(IT)^2L^{-3}L^2)/((LT^{-1})^2\times M\times T) = L^{-1},$$

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which is the inverse of a length.

The quantity X specifically represents an absorption coefficient, usually referred to as α. Absorption is commonly used to express the attenuation of the intensity, I, of an electromagnetic wave propagating in a dissipative medium, according to Beer-Lambert law, $I = I_0 e^{-\alpha z} = I_0 e^{-z/z_0}$, where z is the propagation direction. Thus, one can see that α represents the inverse of the length over which the electromagnetic wave intensity reduces by a factor $1/e$. In semiconductors, α is commonly expressed in $\mathrm{cm^{-1}}$.

Exercise 3

Recalling that $\boldsymbol{F}$ is expressed in $\mathrm{V.m^{-1}}$ when using SI units, we get $[\boldsymbol{F}] = (\mathrm{ML^2T^{-3}I^{-1}})\times\mathrm{L^{-1}}$, and thus one has $[\mu] = (\mathrm{LT^{-1}})/((\mathrm{ML^2T^{-3}I^{-1}})\times\mathrm{L^{-1}}) = \mathrm{L^2}\times\mathrm{ML^2T^{-3}I^{-1}}\times\mathrm{T^{-1}}$ whose units will hence be $\mathrm{m^2\ V^{-1}\ s^{-1}}$.
The typical electric fields applied to semiconductor materials are on the order of 10^4-10^7 $\mathrm{V.m^{-1}}$, resulting in velocities on the order of 10^3-10^5 $\mathrm{m.s^{-1}}$. The resulting free carrier mobilities are thus on the order of 10^7 $\mathrm{m^2\ V^{-1}\ s^{-1}}$. However, free carrier mobilities are usually expressed in $\mathrm{cm^2\ V^{-1}\ s^{-1}}$.

Exercise 4

One has:
$[\tau_{rad}] = (\mathrm{M}\times(\mathrm{LT^{-1}})^3\times(\mathrm{ML^2T^{-2}}\times\mathrm{T})^2\times(\mathrm{IT})^2\times(\mathrm{MLT^{-2}})^{-1}\times\mathrm{L^{-2}})/((\mathrm{IT})^2\times(\mathrm{ML^2T^{-2}})^2) = \mathrm{T}$,

which is indeed corresponding to a time.

Exercise 5

a) $[a_0] = ((\mathrm{IT})^2\times(\mathrm{MLT^{-2}})^{-1}\times\mathrm{L^{-2}}\times(\mathrm{ML^2T^{-2}}\times\mathrm{T})^2)/((\mathrm{IT})^2\times\mathrm{M}) = \mathrm{L}$, and we get $a_0 \sim 5.3\times 10^{-11}$ m = 53 pm.
$[Ry] = ((\mathrm{IT})^4\times\mathrm{M})/(((\mathrm{IT})^2\times(\mathrm{MLT^{-2}})^{-1}\times\mathrm{L^{-2}})^2\times(\mathrm{ML^2T^{-2}}\times\mathrm{T})^2) = \mathrm{ML^2T^{-2}}$, which then leads to $Ry \sim 2.18\times 10^{-18}$ J~13.6 eV.

b) In a bulk semiconductor material such as GaAs, electrons are much more weakly bound to their parent donors. The effective Rydberg energy reduces from 13.6 eV to < 10 meV (~7.7 meV in the present case), while the effective Bohr radius now extends over several lattice constants (on the order of ~10 nm). Such donors tend to be ionized at room temperature ($k_B T$ ~ 26 meV), where thermal energy is sufficient to let the electron overcome the binding/ionization energy and contribute to carrier transport.